# A RELATIVISTIC ANALYSIS OF CLOCK SYNCHRONIZATION

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ABSTRACT

The relativistic conversion between coordinate time and atomic time is reformulated to allow simpler time calculations relating analysis in solar-system barycentric coordinates (using coordinate time) with Earth-fixed observations (measuring "earth-bound" proper time or atomic time.) After an interpretation of terms, this simplified formulation, which has a rate accuracy of about $10^{-15}$, is used to explain the conventions required in the synchronization of a world-wide clock network and to analyze two synchronization techniques--portable clocks and radio interferometry. Finally, pertinent experiment tests of relativity are briefly discussed in terms of the reformulated time conversion.

## INTRODUCTION

In the relativistic analysis of very long baseline radio interferometry (VLBI) data as well as spacecraft and planetary radiometric data, primary calculations are often most conveniently made in terms of non-rotating coordinates that have the solar system barycenter as an origin. However, measurements in these applications are usually made by Earth-fixed observers. Consequently, such analyses usually involve a relativistic time conversion (e.g. Moyer 1971) relating solar-system barycentric calculations (using coordinate time) with Earth-fixed observations (measuring atomic time). In this article, this time conversion, including all relevant speed and potential effects, is reformulated in order to facilitate both interpretation and analysis in these applications. After an interpretation of terms, the reformulated equation, which has a rate accuracy of about $10^{-15}$, is used to consider the synchronization conventions associated with a world-wide clock network.

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Since clock stabilities have begun to routinely enter a relativistically significant range ($10^{-12}$ to $10^{-13}$), a discussion of such conventions is presently more than an academic exercise. Two synchronization techniques, portable clocks and VLBI, are analyzed in terms of the simplified time equation. Finally, pertinent experimental tests of relativity are briefly discussed.

## II. REFORMULATION OF THE TIME CONVERSION

In this section, the time conversion between atomic time and coordinate time is reformulated by means of several approximations in order to cast it in a form that is more convenient for most applications. We will assume that, with appropriate rate and epoch specifications, the time reading of an atomic clock will, within the known stability limitations of the clock, be equal to proper time as calculated on the basis of the spacetime metric. Calculations will be made in terms of spacetime coordinates that are non-rotating and have the solar system barycenter as an origin. We will assume that the worldlines of the clocks in terms of these coordinates are known with sufficient accuracy. Coordinate time will be denoted by t and the proper time of the $j^{th}$ earth-bound clock by $\tau_j$.

In this analysis, approximations will be guided by the following considerations. Current, relatively well-developed oscillator technology (H-maser standards) can, at best, provide clocks with a long-term stability no better than $\Delta f/f \approx 10^{-15}$. Because of this instrumental limitation on time measurement, theoretical rate corrections ($d\tau/dt$) of the order of $10^{-15}$ or less will not be retained in the following analysis.

If one retains the most significant terms (i.e., the terms that lead to clock rate corrections greater than $10^{-15}$) in the n-body metric tensor (e.g., Misner, Thorne and Wheeler 1973), then the resulting weak-field approximation to the differential equation relating coordinate time with proper time along the clock's worldline is given by the well-known expression (e.g. Moyer 1971):

$$\frac{d\tau_j}{dt} = 1 - \frac{\dot{\vec{y}}_j \cdot \dot{\vec{y}}_j}{2c^2} + \frac{\phi(\vec{y}_j)}{c^2} \tag{1}$$

where $\vec{y}_j$ and $\dot{\vec{y}}_j$ are the earth-bound clock position and velocity* given as a function of coordinate time and where $\phi(\vec{y}_j)$ is the total Newtonian gravitational potential at point $\vec{y}_j$. In this expression,

*A dot will denote differentiation with respect to coordinate time.

the $\dot{\vec{y}}_j \cdot \dot{\vec{y}}_j$ term corresponds to special relativistic time dilation due to clock speed and the $\phi/c^2$ term corresponds to the general relativistic gravitational redshift. Note that, in this expression and in the following anaylsis, we treat the spacelike coordinates as vectors, which is an approximation of adequate accuracy under the weak-field assumptions.

The position vector $\vec{y}_j$ can be represented as a sum of a vector $\vec{x}_e$ from the solar system barycenter to the Earth's center of mass and a vector $\vec{x}_j$ from the Earth's center of mass to the clock so that

$$\vec{y}_j = \vec{x}_e + \vec{x}_j \tag{2}$$

In addition, the potential $\phi$ can be represented as a sum of two terms:

$$\phi(\vec{y}_j) = \phi_e(\vec{x}_j) + \phi_r(\vec{x}_e + \vec{x}_j) \tag{3}$$

where $\phi_e$ is the potential due to the Earth's mass and where $\phi_r$ is due to the mass of all other solar system bodies. The geopotential $\phi_e$ is very nearly constant for an Earth-fixed clock while the potential $\phi_r$ varies as the clock moves about due to both Earth spin and Earth orbital motion. Substituting these last two expressions in Eq. (1), we obtain

$$\frac{d\tau_j}{dt} = 1 - \frac{v_e^2 + 2\vec{v}_e \cdot \vec{v}_j + \vec{v}_j^{\,2}}{2c^2} + \frac{\phi_e}{c^2} + \frac{\phi_r}{c^2} \tag{4}$$

where $\vec{v}_e$ is the Earth's orbital velocity $(\dot{\vec{x}}_e)$ and $v_j$ is the clock's geocentric velocity $(\dot{\vec{x}}_j)$. The order of magnitude of the various terms in Eq. (4) is as follows:

$\frac{v_e}{c} \approx 10^{-4}$ for Earth orbital speed

$\frac{v_j}{c} \approx 10^{-6}$ for clock geocentric speed

$\frac{\phi_r}{c^2} \approx 10^{-8}$ for gravitational potential at the Earth's orbit

$$\frac{\phi_e}{c^2} \approx 10^{-9} \qquad \text{for geopotential at the Earth's surface}$$

Two terms, $\vec{v}_e \cdot \vec{v}_j$ and $\phi_r$, will now be manipulated* into more useful forms. As shown below, these manipulations lead to a time conversion for Earth-fixed clocks that does not involve an integral over $\vec{x}_j(t)$, the clock's time-varying position relative to the Earth's center of mass.

In order to separate Earth spin and Earth orbital motion, expand $\phi_r$ as follows:

$$\phi_r(\vec{x}_e + \vec{x}_j) \approx \phi_r(\vec{x}_e) + \nabla\phi_r(\vec{x}_e) \cdot \vec{x}_j. \tag{5}$$

It is readily shown that neglected quadratic terms due to the sun and moon are of the order of $10^{-17}$ at one earth radius. If one neglects relativistic terms of the order $10^{-15}$, the gradient-$\nabla\phi_r$ is the acceleration $(\vec{a}_e)$ of the Earth's center of mass so that

$$\phi_r(\vec{x}_e + \vec{x}_j) \approx \phi_r(\vec{x}_e) - \vec{a}_e \cdot \vec{x}_j \ . \tag{6}$$

The time equation can be further modified by means of the identity:

$$\vec{v}_e \cdot \vec{v}_j = \frac{d}{dt}\left[\vec{v}_e \cdot \vec{x}_j\right] - \vec{a}_e \cdot \vec{x}_j \tag{7}$$

After substituting Eqs. (6) and (7) in Eq. (4), and integrating over coordinate time from $t_c$ to $t$, we obtain the expression:

$$\tau_j(t) \doteq \tau_c + t - t_c + \Delta t_s + \Delta t_j \tag{8}$$

where the two relativistic "correction" terms are defined by:

$$\Delta t_s \equiv \frac{1}{2c^2} \int_{t_c}^{t} \left[2\phi_r(\vec{x}_e) - v_e^2\right] dt \tag{9}$$

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*A similar but unpublished analysis has been independently carried out by D. S. Robertson and R. D. Reasenberg at the Department of Earth and Planetary Science at MIT.

$$\Delta t_j \equiv \frac{1}{2c^2} \int_{t_c}^{t} \left[ 2\phi_e(\vec{x}_j) - v_j^2 \right] dt \quad - \frac{\vec{v}_e(t) \cdot \vec{x}_j(t)}{c^2} \tag{10}$$

and where $\tau_c$ is the constant of integration. (It can be easily shown that, in order for all clocks to be synchronized according to the earthbound techniques of Section IV, this constant of integration must be the same for all clocks.) In this expression, we have divided the relativistic terms into two categories: the common terms ($\Delta t_s$) which are the same for all clocks, and the clock-specific terms ($\Delta t_j$) which might be different for each clock. Note that the two acceleration terms produced by the speed and potential terms have canceled. This cancelation in a different formulation has been noted by Hoffman (1961) and described as a manifestation of the equivalence principle for freely-falling geocentric coordinates. In this final form, the orbital and spin motions have been separated, except for the $\vec{v}_e \cdot \vec{x}_j$ term.

For a clock fixed with respect to Earth, the speed $v_j$ and potential $\phi_e(\vec{x}_j)$ vary by no more than about one part in $10^6$ (due, for example, to polar motion, earth tides, crustal motions). Therefore, to adequate approximation, the clock-specific correction for an <u>Earth-fixed clock</u> becomes:

$$\Delta t_j = \frac{2\phi_e(\vec{x}_j) - v_j^2}{2c^2} (t - t_c) \quad - \frac{\vec{v}_e(t) \cdot \vec{x}_j(t)}{c^2} \tag{11}$$

These expressions for the time conversion simplify the analysis in the following section which includes a discussion of terms and synchronization conventions.

## III. INTERPRETATION OF TERMS AND CLOCK SYNCHRONIZATION ANALYSIS

World-wide timekeeping is now accomplished by a network of atomic clocks placed at various locations over the Earth. In this network member clocks are periodically synchronized with a master clock, which is carefully maintained at a fixed location. ("Master clock" in practice is the average time reading of a set of reference atomic clocks. Specific techniques for synchronization will be discussed in Section V on the basis of the relativistic time conversion.) Most synchronization work, based on the principles of classical physics, presently assumes that clocks, once synchronized in time and rate, will continue to indicate the same time, within instrumental accuracy, wherever they are moved on the Earth's surface. However, relativistic analysis, such as Eq. (8), indicates that classical assumptions may not be adequate if clock accuracies surpass the μs level in time and the $10^{-12}$ level in rate. That is, sufficiently accurate clocks can lose synchronization due to relativistic effects if they are separated on the Earth's surface. Consequently, an accurate clock network based on relativity theory must take these effects into account.

A relativistic understanding of the synchronization problem is facilitated by the formulation in Eq. (8) which connects coordinate time with the proper time of a given Earth-fixed clock. Even though this equation is not a direct comparison of Earth-fixed clocks, it contains all the information needed to study the synchronization problem, provided the various terms are properly interpreted. The following discussion attempts such an interpretation with emphasis on the establishment of synchronization conventions. Even though some aspects of this discussion are relatively well-known, they have been briefly included, sometimes without reference, for the sake of completeness.

The common correction term $\Delta t_s$ in Eq. (9) contains the factors that cause the same rate offset for all clocks: the speed of the Earth center-of-mass and the "clock-invariant part" of the potential which is located at the Earth center-of-mass. Since this term is common to all clocks in the network, it will not cause a loss of synchronization. That is, this term is not significant in "earth-bound" comparisons of the clocks but is significant when converting between coordinate time and atomic time. In practice, the common term must be modified to account for any conventions affecting overall clock rates. For example, it is convenient to define the second so that, in principle, all clocks run at the same average rate as coordinate time (e.g. Moyer 1971). This rate definition is represented* formally in $\Delta t_s$ by subtracting the time-average rate from the total rate in Eq. (9) as follows:

$$\widetilde{\Delta t}_s \equiv \Delta t_s - \overline{\Delta t}_s \equiv \frac{1}{2c^2}\int_{t_c}^{t} \left[2\Delta\phi_r - \Delta v_e^2\right] dt \tag{12}$$

where

$$\Delta v_e^2 \equiv v_e^2 - \overline{v_e^2}$$

$$\Delta\phi_r \equiv \phi_r - \overline{\phi}_r .$$

This rate adjustment, which is the order of $10^{-8}$, leaves only the nonlinear, principally periodic effects in $\Delta t_s$. (A detailed analysis to

*In the remaining analysis, a tilde over a term will denote that the term has been adjusted according to the synchronization conventions defined in this section.

establish the most appropriate definition for this time average and to model the most important time-varying residual terms is beyond the scope of this paper.) Even though these residual time-varying effects do not cause loss of synchronization between Earth-fixed clocks, they must still be included in conversions between proper time and coordinate time. For example, the predominant effect, orbital eccentricity, has an integrated amplitude of approximately 2 ms and an annual period (e.g. Moyer 1971).

The clock-specific correction $\Delta t_j$ in Eq. (11) can lead to a synchronization loss between Earth-fixed clocks. This correction can be subdivided into two categories of time dependence for Earth-fixed clocks: linear and periodic.

In the first category, the linear term, $\left[\phi_e - v_j^2/2\right]$ $(t - t_c)$, is a rate correction based on clock geopotential and speed relative to the Earth's center-of-mass. This term corresponds to the conventional redshift due to geopotential since its coefficient is essentially the effective geopotential at point $\vec{x}_j$ as classically observed at the Earth-fixed clock (Cocke 1966). That is, the gradient of $v_j^2/2 - \phi_e$ gives the sum $(\vec{g})$ of the earth-spin "centrifugal force" and the earth's gravitational attraction at that point. Since mean sea level represents, to good approximation, a surface of constant effective geopotential, all Earth-fixed clocks at mean sea level should run at approximately the same rate without relativistic corrections. For two arbitrary Earth-fixed clocks, the differential rate correction can be approximately calculated on the basis of differential altitude by the formula $g\Delta h$, which predicts that the rate correction changes by approximately $1.1 \times 10^{-13}$ per kilometer of differential altitude. (For airborne or orbiting clocks, it is readily shown that time-varying differential rate corrections of the order of $10^{-9} - 10^{-12}$ are possible.)

In the second category, the periodic term $\vec{v}_e(t)\cdot\vec{x}_j(t)$ is never greater than 2 μs and is essentially diurnal since $\vec{v}_e$ changes very little over one day. This term corresponds to the special relativity clock synchronization correction that accounts for the fact that simultaneous events in one frame (a "solar system frame") are not necessarily simultaneous in a frame (a "geocentric frame") passing by with velocity $\vec{v}_e$. Consequently, it is of significance in conversions between Earth-fixed time and coordinate time but is not present in "Earth-bound" comparisons between Earth-bound clocks. This assertion is supported analytically by the fact that the periodic term "changes" to match another clock if the two clocks are brought together on Earth.

The following conventions regarding synchronization are designed to accommodate these clock-specific terms. Since the linear terms can lead to gross disagreements between clocks over long time periods, they will be removed, either explicitly or implicitly, by making appropriate location-dependent definitions of clock rate. For such rate adjustments to be significant, the stability of the clocks in the network must, of course, be significantly better than a typical rate correction of roughly $10^{-13}$. In principle, these corrections could be applied by means of explicit on-site rate adjustments based on a fundamental physical process. For example, at each location a second could be set equal to a particular altitude-dependent number of cycles (to more than 13 decimal places) on a cesium beam frequency standard where the cycle-count differential between altitudes would be based on the differential in effective potential. Since these rate adjustments are of the order of $10^{-13}$, the oscillators would necessarily have to be capable of independent (absolute) calibration at a few parts in $10^{-14}$ or better (in addition to the similar stability requirement). Unfortunately, routine calibrations at this level are not feasible at present. In practice, this rate adjustment will be implicitly applied in a differential sense whenever a world-wide clock network is kept in time synchronization. For example, as in the present system, a "master clock" would be utilized, at a given location, to define the second and maintain a reference time. Other clocks over the world would then be forced into synchronization by means of "earth-bound" synchronization techniques (see Section V). Since the synchronization process prevents clock divergence, the appropriate differential rate correction will be implicitly applied without recourse to relativistic calculations.

Since the periodic term $\vec{v}_e(t) \cdot \vec{x}_j(t)$ does not affect the synchronization of Earth-bound clocks, it is not of consequence in the establishment of a synchronization convention.

By modifying Eq. (8) according to the definitions and conventions described above, one obtains a standardized conversion for Earth-fixed clock j:

$$\tilde{\tau}_j(t) = \tau_c + t - t_c + \frac{1}{2c^2}\int_{t_c}^{t} \left[2\Delta\phi_r(\vec{x}_e) - \Delta v_e^2\right] dt - \frac{\vec{v}_e(t) \cdot \vec{x}_j(t)}{c^2} \qquad (13)$$

Note that the time equation no longer involves an integral over clock coordinates but only over coordinates for the Earth's center-of-mass. Therefore, relative to the original formulation, time calculations are much simpler.

In summary, with the conventions outlined above, the network clocks would be given selected initial times (at coordinate time $t_c$) and the same average rate (i.e. $\overline{d\tau/dt} = 1$). With these conventions, the clock network could be kept in synchronization according to Earth-bound observers by means of two synchronization methods now in use. These two techniques, portable clocks and VLBI, will be discussed in Section V in terms of these synchronization conventions.

## IV. VLBI TIME DELAY

Radio interferometry holds great promise as a technique for the accurate synchronization of a world-wide clock network as well as for the accurate measurement of a variety of geophysical and astronomical phenomena [Shapiro and Knight 1970]. In this section, the primary component of the interferometric delay observable, the geometric delay, is calculated through the use of the reformulated time equation of the last section. The purpose of this derivation is to clarify the origin of the various terms in the geometric delay in preparation for a discussion of clock synchronization in Section V and experimental tests in Section VI.

The VLBI geometric delay is readily calculated using Eq. (13) as follows. Suppose that radio waves emitted by a distant source are observed by two Earth-fixed antennas. Let a given wavefront reach antenna 1 at coordinate time t and antenna 2 at t'. According to the two antenna teams, the wavefront arrives at $\tilde{\tau}_1(t)$ at antenna 1 and time $\tilde{\tau}_2(t')$ at antenna 2. When the two antenna teams compare arrival times, they will measure the "geometric" delay:

$$\tau_g(t) \equiv \tilde{\tau}_2(t') - \tilde{\tau}_1(t) \; . \tag{14}$$

We have assumed that instrumental and transmission media delays have been removed. In addition, we have assumed that the antenna clocks have been synchronized according to the conventions leading to Eq. (13). (A loss of synchronization would, of course, appear as an additive term in the measured delay.)

Since $|t' - t|$ is less than 30 ms for Earth-fixed antennas, the terms containing t' can be expanded about t to yield:

$$\tau_g(t) = \tilde{\tau}_2(t) - \tilde{\tau}_1(t) + \dot{\tilde{\tau}}_2(t)\,(t' - t) \tag{15}$$

$$= t' - t + \frac{1}{c^2}\left[\Delta\phi_r(\vec{x}_e) - \Delta v_e^2/2 - \vec{v}_e \cdot \vec{v}_2\right](t' - t) - \frac{\vec{v}_e(t) \cdot \vec{b}(t)}{c^2} \; , \tag{16}$$

where the baseline $\vec{b}$ equals $\vec{x}_2 - \vec{x}_1$. In this expression, we have neglected an $\vec{a}_e \cdot \vec{x}$ term and terms of order higher than the first in $t' - t$ with negligible loss of accuracy. Note that the geometric delay is equal to the "coordinate time delay", $t' - t$, plus time conversion corrections of two types. The first type is a "time dilation" correction, consisting of three terms proportional to $t' - t$. It is easily demonstrated that these terms are less than 20 psec (0.6cm) in magnitude. Consequently, these corrections are of marginal importance for even the most ambitious VLBI applications.

The second correction category, which corresponds to the clock synchronization correction (or aberration correction) found in a special relativity treatment, can be estimated as follows:

$$\frac{\vec{v}_e \cdot \vec{b}}{c^2} < 10^{-4} \cdot \frac{12000}{c} = \frac{1.2 \text{ km}}{c} = 4 \ \mu\text{sec} \qquad (17)$$

Since $\vec{v}_e$ changes very little over a day, this term exhibits essentially diurnal time variations. In time delay calculations, this large correction must be treated very precisely.

Upon to this point, the coordinate time delay $t' - t$ has been treated in a general fashion and could denote any two events occurring near the earth. Since this section is primarily concerned with the relativistic conversion of a given coordinate time delay to proper time observations, a general discussion of delay calculations, including all factors, will not be attempted. However, as an example, the time delay for a very distant, fixed source (specifically, a compact extragalactic radio source) will be approximately derived in preparation for Section V and VI.

The geometric delay for an extragalactic source can be derived by first calculating the coordinate time delay and then transforming to antenna observers. We will give the signal a plane-wave representation that ignores transmission media and general relativity effects. The coordinate time delay for a plane wave is then easily shown to be given by

$$t' - t = - \frac{\hat{s} . \vec{b}}{c \left[ 1 + s \cdot (\vec{v}_e + \vec{v}_2)/c \right]} , \qquad (18)$$

where $\hat{s}$ is a unit vector in the direction of the radio source relative to the solar system barycenter. The observed time delay is then obtained by inserting this expression into the time conversion, Eq. (16). All quantities in this expression are evaluated at coordinate time t, the time the wave front reaches antenna 1.

As an alternate but instructive approach, the geometric delay can be approximately derived to order v/c relative to a geocentric frame (e.g. Thomas 1972). In that derivation, the $\vec{v}_e \cdot \vec{b}$ term enters the delay as a result of the aberration correction to the source direction. As indicated by the two derivations, this large term can be viewed in two ways. For Earth-bound observers, it is a geometric correction applied to the position of the source. In contrast, relative to solar-system barycentric coordinates, it can be viewed as a time correction corresponding to a loss of synchronization between Earth-fixed clocks in the special relativity limit.

V. CLOCK SYNCHRONIZATION TECHNIQUES

This section will show how two synchronization techniques, portable clocks and VLBI, can be used to synchronize a world-wide clock network according to the synchronization conventions defined in Section III. The portable clock technique will be discussed first.

In the present world-wide timekeeping network, member clocks at various locations around the world are periodically resynchronized with the "master clock" by comparing them with a portable clock that is carried to each location. Before and after each trip, the portable clock is synchronized on-site with the "master clock". In this manner, a world-wide network of clocks can be placed in synchronization at the level (presently 1-10 μsec for routine applications) allowed by the instrumental and transportation stability of clocks involved.

Let a portable clock be synchronized with the master clock at coordinate time, $t = t_0$. Then let the portable clock follow path[1] $\vec{x}_p(t)$ over the Earth to some member of the clock network. (Note that $\vec{x}_p(t)$ and $\vec{v}_p(t)$ contain Earth-spin as well as clock transportation effects.) After the portable clock has reached the member clock j at time t', the clock-specific correction for the portable clock [see Eq. (10) but subtract the constant master-clock rate] will be

$$\Delta t_p = \frac{1}{2c^2} \int_{t_0}^{t} \left[ 2\phi_e(\vec{x}_p) - v_p^2 - 2\phi_e(x_m) + v_m^2 \right] dt - \frac{\vec{v}_e(t') \cdot \vec{x}_j(t')}{c^2} , \qquad (19)$$

[1]Relative to the Earth center-of-mass.

where the subscripts p and m denote portable and master clocks respectively. (We have not included the other terms in Eq. (8) in this discussion since they are common to all clocks and do not affect synchronization.) The integral term in this expression accounts for the fact that the master clock rate adjustment (passed on to the portable clock during synchronization with the master clock) will not suppress the $2\phi_e - v_p^2$ integral for the portable clock once it starts its journey and changes its geocentric position and speed. Furthermore, the term $\vec{v}_e(t') \cdot \vec{x}_j(t')$ represents the value for the correction term $\vec{v}_e \cdot \vec{x}_p$ after the portable clock reaches the member clock ($\vec{x}_p \to \vec{x}_j$).

According to the synchronization conventions established in Section III, the portable clock-member clock comparison must be handled as follows. The desired value for the clock-specific term for the member clock is given by

$$\widetilde{\Delta t}_j = - \frac{v_e(t') \cdot x_j(t')}{c^2} \quad . \tag{20}$$

Thus, comparing Eq. (19) and Eq. (20), we see that the portable clock must be corrected to account for the speed-potential integral that has accumulated in transit:

$$\Delta\tau_p = \widetilde{\Delta t}_p - \widetilde{\Delta t}_j \tag{21}$$

$$= \frac{1}{2c^2} \int_{t_o}^{t'} \left[ 2\phi_e(\vec{x}_p) - v_p^2 - 2\phi_e(\vec{x}_m) + v_m^2 \right] dt \tag{22}$$

For one day transit times, this correction can be of the order of $10^{-12} \times 10^5$ s = 100 ns. Further, the portable clock rate will differ from the conventional rate for site j by

$$\frac{2\phi_e(\vec{x}_j) - v_j^2 - 2\phi_e(\vec{x}_m) + v_m^2}{2c^2} \approx \frac{g\Delta h}{c^2} \quad , \tag{23}$$

so that the clock rate comparisons must include this correction factor. Thus, we see that, during transit, the periodic term $\vec{v}_e \cdot \vec{x}_p$ changes into the appropriate value while the "rate term" loses its adjustment and must be corrected.

It is interesting to note that the integral contained in Eq. (22) is essentially the theoretical time gain predicted by Hafele and Keating for their Earth-circumnavigation experiment (Hafele and Keating 1972), in which they measured the synchronization loss (relative to a stationary master clock) of atomic clocks flown around the world. In that paper, theoretical calculations only considered geocentric speed and geopotential effects. With a more general approach, the present formulation indicates that this integral is the total time gain, provided one can neglect rate terms less than about $10^{-15}$. Thus, the warning by Hafele and Keating that effects of the sun and moon might perhaps not be entirely negligible appears to be unwarranted for clock stabilities worse than about $10^{-15}$.

Clock synchronization by means of VLBI is conceptually, if not operationally, straightforward. For a given natural source, the time delay is measured between two antennas and appropriately corrected for transmission media and instrumental delays. The resulting delay should be equal to the geometric delay calculated according to Eq. (16). (We assume here that geophysical and astrometric quantities are known with sufficient accuracy.) Any difference between the measured delay and the calculated delay represents the synchronization loss between antenna clocks. In this manner, a world-wide system of clocks could be synchronized at interferometer accuracies, which are expected to reach about 0.1 nsec for future well-developed VLBI systems (Shapiro and Knight 1970).

## VI. DISCUSSION OF PERTINENT RELATIVITY EXPERIMENTS

In a treatment of this nature, it is of interest to discuss pertinent tests of relativity in terms of the reformulated time conversion. As indicated in Section III, only the "geocentric" term, the integral in Eq. (8), will be evident in "earth-bound" comparisons of Earth-bound clocks. Contingent on instrumental feasibility, two basic types of Earth-bound clock experiments can be used to test the presence of this effect: moving clocks and Earth-fixed clocks. It is interesting to note that, while a moving clock experiment generally involves an integrated synchronization loss due to time-varying speed and potential, the relativistic effect for Earth-fixed clocks would, to first approximation, only consist of a constant rate offset due to a constant differential in geopotential. Consequently, an Earth-fixed experiment would essentially produce, to first order, a measurement of the conventional gravitational redshift, as first measured by Pound and Rebka (1960) and later but more accurately by Pound and Snider (1964). In those experiments, precise nuclear resonance measurements based on the Mössbauer effect determined the apparent frequency shift of $\gamma$-rays from the 14.4 kev transition in $Fe^{57}$ where the local geopotential difference was established with a 22.5 m altitude differential.

In order for an Earth-fixed clock experiment to equal the 1% accuracy of the Pound-Snider experiment, a typical rate differential of roughly $10^{-13}$ (for a 1 km altitude differential) would have to be measured with an accuracy of $10^{-15}$, which would, of course, require clock stability and accuracy at $10^{-15}$. Except perhaps at standards laboratories, this clock rate calibration requirement would probably be the most difficult aspect of such an Earth-fixed clock experiment. One of the possible methods for measuring the synchronization loss in such an experiment would be the VLBI technique, which would have the advantageous option, for portable antennas, of making very precise synchronization measurements between standard clocks while they are carefully maintained in the existing ideal environments of two possibly widely separated standards laboratories. For example, with a VLBI clock synchronization accuracy of 0.1 nsec, a rate - differential sensitivity of roughly $10^{-15}$ would be obtained with about 24 hours of data. Unfortunately, even though the $\gamma$-ray resonance and clock techniques are significantly different, the current, established level in clock technology will not allow an Earth-fixed-clock measurement of the geopotential redshift that is significantly more accurate than the Pound-Snider experiment.

In general, the relativistic rate effect can be larger for moving clocks than for Earth-fixed clocks, potentially by as much as three orders of magnitude. One moving-clock experiment, involving airborne clocks (Hafele and Keating 1972), has already been carried out with positive results. In that experiment, the synchronization losses (273 nsec in the maximum case) relative to an Earth-fixed standard clock were observed to be in agreement with theoretical estimates, with an overall uncertainty of about 20 nsec. Another moving-clock experiment, presently in preparation (Kleppner et al, 1970; Vessot and Levine, 1971), would attempt to measure the time-varying relativistic rate differential between an Earth-orbiting clock and an Earth-fixed clock. Measurements of rate variations as large as $6 \times 10^{-10}$ with an overall accuracy of 1 part in $10^5$ are predicted for that experiment.

## VII. SUMMARY

In the preceding sections, a reformulation of the relativistic time equation has simplified interpretation of the various effects entering the conversion between coordinate time and Earth-bound proper time (atomic time). Based on this analysis, the conventions and techniques involved in the synchronization of a world-wide clock network have been investigated. In addition, the new formulation has simplified a relativistic analysis of the "geometric delay" measured in VLBI applications. Finally, a brief discussion has been devoted to "Earth-bound" relativity experiments that are relevant to the reformulation.

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